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New Efficient Recursive Algorithms for Estimating the State, Controllability and Observability of Large Power Systems

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1 INTRODUCTION

Controllability and observability of power system are key properties when addressing many challenges for regulation and control. Evaluation of these properties in Tempe transients sufficient hard in the algorithmic aspect. Therefore, developing effective computational procedures for the state estimation, observability and controllability power system as a multivariable dynamical system is quite relevant.

Consider large power system given by the equations

$$(1) \quad \dot{x}(t) = Ax(t) + Bu(t), \quad y = Cx(t),$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^r$ is the control, $y \in \mathbb{R}^m$ is the output, $\text{rank } B = r$, $\text{rank } C = m$.

By virtue of duality principle, the algebraic methods for determining controllability and observability of power system (1) are similar [1]. Following this principle, let us focus on the controllability analysis problem and extend final results onto the observability analysis problem.

Kalman rank controllability criterion [3, 4] and Popov-Belevich-Hautus modal criterion (PBH-test) [5] are more frequently used to analyze controllability of system (1).

In some cases, use of rank criterion results in false conclusion on system uncontrollability because of increase of condition number of considered matrix [6, 7]. In this case, PBH-test is more effective, although to apply it, one need compute eigenvalues of analyzed system. Often it also leads to ill-conditioned problem.

In authors' humble opinion, the band criteria described in [6, 7] belong to the most general approach that allows to analyze controllability of system (1) from common point of view. However, in spite of all advantages of these criteria [6, 7], their major drawback consists in significant increase of band controllability matrix dimensions. It have been pointed out by well-known French scientist M. Malabre in his comments to paper [7].

In response to observation of M. Malabre, some recursive tests (algorithms) for controllability and observability analysis on the basis of the band criteria of controllability and observability [6, 7] are presented. These tests allow significant reduction of computation cost for system (1) with large dimensions.

2 BAND CONTROLLABILITY CRITERION FOR POWER SYSTEM

Let us present the band controllability criterion for system (1) [6, 7].

Criterion 1. Power system (1) is fully controllable if and only if the band controllability matrix

$$(2) \quad \mathbf{C}_{\text{band}} = \begin{pmatrix} B_L^\perp A & 0 & 0 & \dots & 0 \\ B_L^\perp & B_L^\perp A & 0 & \dots & 0 \\ 0 & B_L^\perp & B_L^\perp A & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & B_L^\perp \end{pmatrix} \in \mathbb{R}^{(n-r+1)(n-r) \times (n-r+1)(n-1)}$$

has full row rank¹.

Here $B_L^\perp$ is the so called *left annihilator* of maximum rank of the matrix B , being the left maximum solution of homogeneous algebraic equation [1, 8, 9]

$$(3) \quad XB = 0,$$

that is, $\text{rank } X = n - r$.

The left annihilators (sometimes called left zero divisors) are not unique and exist for any numeric matrix with incomplete row rank [10, 11]. All the set of left zero divisors of the matrix B (not necessarily with full row rank) can be obtained from $B_L^\perp$ via left multiplication by some numeric matrix μ of compatible dimension with arbitrary elements. Thus, all the solution set for equation (3) can be obtained as

$$(4) \quad X = \mu B_L^\perp, \quad \mu \neq 0.$$

If the matrix B is full in rows (does not have linearly dependent rows), then it does not have a left zero divisor or it equals to zero, and equation (3) does not have a nontrivial solution.

For matrices with incomplete column rank, the symmetric statements hold true. Such the matrices always have the right zero divisors, including ones of maximum rank.

According to presented controllability criterion on the base of band matrix (2), power system (1) is fully controllable if and only if

$$(5) \quad \text{rank } \mathbf{C}_{\text{band}} = (n - r + 1)(n - r).$$

Drawback of criterion (5) consists in increased dimensions of matrix (2) in comparison with R. Kalman controllability criterion that supposes rank computation for the following matrix:

$$(6) \quad \text{rank } \mathbf{C} = \text{rank} \begin{pmatrix} B & AB & A^2B & \dots & A^{n-r}B \end{pmatrix} = n$$

with smaller dimensions.

The next sections presents the recursive controllability test properly allowing significant reduction of computation cost.

3 RECURSIVE CONTROLLABILITY TEST FOR POWER SYSTEM

Let us formulate the following recursive controllability test for system (1).

Step 0. Find the maximum solution $B_L^\perp$ of equation $B_L^\perp B = 0$. If $B_L^\perp = 0$, then the test is complete². One can make a conclusion on system controllability.

Step 1. Find the maximum solution $(L_1^{(1)} \vdots L_2^{(1)})$ of equation

$$(7) \quad \begin{pmatrix} L_1^{(1)} & L_2^{(1)} \end{pmatrix} \begin{pmatrix} B_L^\perp A \\ B_L^\perp \end{pmatrix} = 0.$$

¹ Denote, that for matrix (2), the number of rows never exceeds the number of columns, because $n - r \leq n - 1$.

² This statement follows directly from R. Kalman criterion and the band criterion. In second case, with $B_L^\perp = 0$, the band matrix is zero and always has maximum rank that equals to zero.

If $\begin{pmatrix} L_1^{(1)} \\ L_2^{(1)} \end{pmatrix} = 0$, then the test is complete (power system is controllable).

Step 2. Find the maximum solution $\begin{pmatrix} L_1^{(2)} \\ L_2^{(2)} \end{pmatrix}$ of equation

$$\begin{pmatrix} L_1^{(2)} \\ L_2^{(2)} \end{pmatrix} \begin{pmatrix} L_2^{(1)} \\ L_1^{(1)} \end{pmatrix} = 0.$$

If $\begin{pmatrix} L_1^{(2)} \\ L_2^{(2)} \end{pmatrix} = 0$, then the test is complete (power system is controllable).

Step k. Find the maximum solution $\begin{pmatrix} L_1^{(k)} \\ L_2^{(k)} \end{pmatrix}$ of equation

$$(8) \quad \begin{pmatrix} L_1^{(k)} \\ L_2^{(k)} \end{pmatrix} \begin{pmatrix} L_2^{(k-1)} \\ L_1^{(k-1)} \end{pmatrix} = 0.$$

If $\begin{pmatrix} L_1^{(k)} \\ L_2^{(k)} \end{pmatrix} = 0$, then the test is complete (power system is controlled).

For a controllable system, the test will cease not later than at $(n-r+1)$ -th step. Besides, from aforesaid reasoning it follows that nondecreasing matrix rank in the sequence

$$(9) \quad \begin{pmatrix} L_2^{(k-1)} \\ L_1^{(k-1)} \end{pmatrix}, \begin{pmatrix} L_2^{(k)} \\ L_1^{(k)} \end{pmatrix}$$

can be considered as *stop condition* for the controllability recursive test.

Also denote, that in the matrices

$$B, \begin{pmatrix} B_L^\perp A \\ B_L^\perp \end{pmatrix}, \begin{pmatrix} L_2^{(1)} \\ L_1^{(1)} \end{pmatrix}, \dots, \begin{pmatrix} L_2^{(k)} \\ L_1^{(k)} \end{pmatrix}, \dots$$

one can do any row permutation, since this operation type belongs to the equivalent transformations.

4 RECURSIVE OBSERVABILITY TEST FOR POWER SYSTEM

By virtue of controllability and observability problems duality for power system, in this section the band observability criteria and corresponding recursive observability tests are presented without detailed comments.

The observability notion is considered in respect of power system (1) with output variable vector (system output) $y(t) = Cx(t)$.

Let us present the band observability criterion [6, 7] and the recursive observability test.

Criterion 2. Power system (1) is fully observable if and only if the band observability matrix

$$(10) \quad \mathbf{O}_{\text{band}} = \begin{pmatrix} AC_R^\perp & C_R^\perp & 0 & \dots & 0 \\ 0 & AC_R^\perp & C_R^\perp & \dots & 0 \\ 0 & 0 & AC_R^\perp & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & C_R^\perp \end{pmatrix} \in \mathbb{R}^{(n-m+1)(n-1) \times (n-m+1)(n-m)}$$

has full column rank [6].

Here $C_R^\perp$ is the so called *left annihilator (right zero divisor)* of maximum rank of the matrix C , being the right maximum solution of homogeneous algebraic equation [1, 8]

$$(11) \quad CY = 0,$$

where $\text{rank } Y = n - m$.

Let us formulate the following recursive observability test for system (1).

Step 0. Find the maximum solution $C_R^\perp$ of equation $CC_R^\perp = 0$. If $C_R^\perp = 0$, then the test is completed.

One can make a conclusion on observability of the considered power system.

Step 1. Find the maximum solution of equation

$$\left(AC_R^\perp \quad C_R^\perp \right) \begin{pmatrix} R_1^{(1)} \\ \vdots \\ R_2^{(1)} \end{pmatrix} = 0.$$

If $\begin{pmatrix} R_1^{(1)} \\ \vdots \\ R_2^{(1)} \end{pmatrix} = 0$, then the test is completed (the system is observable).

Step 2. Find the maximum solution of equation

$$\left(R_2^{(1)} \quad R_1^{(1)} \right) \begin{pmatrix} R_1^{(2)} \\ R_2^{(2)} \end{pmatrix} = 0.$$

If $\begin{pmatrix} R_1^{(2)} \\ R_2^{(2)} \end{pmatrix} = 0$, then the test is completed (the considered power system is observable).

Step k. Find the maximum solution of equation

$$\left(R_2^{(k-1)} \quad R_1^{(k-1)} \right) \begin{pmatrix} R_1^{(k)} \\ R_2^{(k)} \end{pmatrix} = 0.$$

If $\begin{pmatrix} R_1^{(k)} \\ R_2^{(k)} \end{pmatrix} = 0$, then the test is completed (the considered power system is observable).

By analogy with the controllability test, nondecreasing rank of matrices $\left(R_2^{(k-1)} \quad R_1^{(k-1)} \right)$, $\left(R_2^{(k)} \quad R_1^{(k)} \right)$ can be used as the stop condition for the recursive observability test.

5 ELECTRIC POWER SYSTEM OBSERVABILITY ANALYSIS

Using the developed test, let us analyze observability of electric power system including two electric power stations operating with common consumer buses. The mathematical model of this system is given in parametric form [12]

$$(12) \quad A = \begin{pmatrix} -G_1 M_1^{-1} & 0 & -M_1^{-1} & 0 & 0 & M_1^{-1} & 0 & 0 \\ 0 & -G_2 M_2^{-1} & M_1^{-1} & 0 & 0 & 0 & 0 & M_2^{-1} \\ T_{12} & -T_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ -E_1 T_{g_1}^{-1} & 0 & 0 & -T_{g_2}^{-1} & 0 & 0 & 0 & 0 \\ 0 & -E_2 T_{g_2}^{-1} & 0 & 0 & -T_{g_2}^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & T_{t_1}^{-1} & 0 & -T_{t_1}^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & T_{t_2}^{-1} & 0 & -T_{t_2}^{-1} & 0 \\ 0 & 0 & 0 & 0 & -2D_2^{-1} & 0 & 2(D_2^{-1} + T_{t_2}^{-1}) & -2D_2^{-1} \end{pmatrix},$$

$$(13) \quad C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

In this case, the state vector $x(t) \in \mathbb{R}^8$ includes the following elements: $x_1(t)$, $x_2(t)$ — deviations of electric current frequency of the first and second electric power stations, correspondingly, $x_3(t)$ — deviation of electric power, $x_4(t)$, $x_5(t)$, $x_8(t)$ — signals of controllers, $x_6(t)$, $x_7(t)$ — deviations of mechanical power for the first and second electric power stations, correspondingly.

Step 0. Find the right zero divisor $C_R^\wedge$ of matrix (13):

$$(14) \quad C_R^\perp = \begin{pmatrix} 0_{3 \times 5} \\ E_5 \end{pmatrix} \neq 0.$$

Since (14) is the nonzero matrix, go onto the next step.

Step 1. Find the maximum solution $\begin{pmatrix} R_1^{(1)} \\ R_2^{(1)} \end{pmatrix}$ of equation

$$(15) \quad (AC_R^\perp \ : \ C_R^\perp) \begin{pmatrix} R_1^{(1)} \\ R_2^{(1)} \end{pmatrix} = 0,$$

where

$$(16) \quad AC_R^\perp = \begin{pmatrix} 0 & 0 & M_1^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_2^{-1} \\ 0 & 0 & 0 & 0 & 0 \\ -T_{g_2}^{-1} & 0 & 0 & 0 & 0 \\ 0 & -T_{g_2}^{-1} & 0 & 0 & 0 \\ T_{t_1}^{-1} & 0 & -T_{t_1}^{-1} & 0 & 0 \\ 0 & T_{t_2}^{-1} & 0 & -T_{t_2}^{-1} & 0 \\ 0 & -2D_2^{-1} & 0 & 2(D_2^{-1} + T_{t_2}^{-1}) & -2D_2^{-1} \end{pmatrix}.$$

We obtain

$$(17) \quad \begin{pmatrix} R_1^{(1)} \\ R_2^{(1)} \end{pmatrix} = \frac{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & T_{t_2} & D_2 + T_{t_2} \\ 0 & 0 & 0,5D_2 \\ -T_{t_1}^{-1} & 0 & 0 \\ -1 & -(D_2 T_{t_2}^{-1} + 0,5) \\ T_{t_1}^{-1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}{\neq 0} \neq 0.$$

Since solution (17) of equation (15) is nontrivial, we can go onto Step 2.

Step 2. Find maximum solution of equation

$$\begin{pmatrix} R_2^{(1)} & R_1^{(1)} \end{pmatrix} \begin{pmatrix} R_1^{(2)} \\ R_2^{(2)} \end{pmatrix} = 0,$$

where the matrix $\begin{pmatrix} R_2^{(1)} & R_1^{(1)} \end{pmatrix}$ is defined in accordance with the block partitioning in expression (17).

We have

$$(18) \quad \begin{pmatrix} R_1^{(2)} \\ R_2^{(2)} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -4T_{t_2}^2(2D_2^2 + 5D_2T_{t_2} + 4T_{t_2}^2)^{-1} \\ 0 \\ (2D_2 + T_{t_2})D_2T_{t_2}(2D_2^2 + 5D_2T_{t_2} + 4T_{t_2}^2)^{-1} \\ 2D_2T_{t_2}^2(2D_2^2 + 5D_2T_{t_2} + 4T_{t_2}^2)^{-1} \end{pmatrix} \neq 0,$$

$$2D_2^2 + 5D_2T_{t_2} + 4T_{t_2}^2 \neq 0.$$

By virtue of (18), go onto Step 3.

Step 3. Find maximum solution of equation

$$(19) \quad \begin{pmatrix} R_2^{(2)} & R_1^{(2)} \end{pmatrix} \begin{pmatrix} R_1^{(3)} \\ R_2^{(3)} \end{pmatrix} = 0,$$

where the matrix $\begin{pmatrix} R_2^{(2)} & R_1^{(2)} \end{pmatrix}$ is defined in accordance with the block partitioning in (18).

Since the matrix

$$(20) \quad \begin{pmatrix} R_2^{(2)} & R_1^{(2)} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & (2D_2 + T_{t_2})D_2T_{t_2}(2D_2^2 + 5D_2T_{t_2} + 4T_{t_2}^2)^{-1} \\ -4T_{t_2}^2(2D_2^2 + 5D_2T_{t_2} + 4T_{t_2}^2)^{-1} & 2D_2T_{t_2}^2(2D_2^2 + 5D_2T_{t_2} + 4T_{t_2}^2)^{-1} \end{pmatrix}$$

has full column rank (for physical parameters $2D_2T_{t_2}^2(1 + 2T_{t_2}(2D_2 + T_{t_2})) \neq 0$), equation (19) has only zero (trivial) solutions. Thus, considered system (12), (13) is observable.

As one can see from the last example, the ability of additional analysis of parametric observability conditions on each step can be placed among the other advantages of the recursive test.

On the other hand, one can make sure, that the observability analysis of pair (12), (13) using Kalman observability criterion

$$\text{rank } \mathbf{O} = \text{rank} \begin{pmatrix} C^T & A^T C^T & (A^T)^2 C^T & \dots & (A^T)^5 C^T \end{pmatrix}_{8 \times 18}$$

results in the intractable problem.

6 COMPUTATION COST

Analyze the computation costs needed to determine the system controllability on the base of the recursive test and Kalman controllability criterion.

The major computation costs of Kalman controllability criterion are determined by involution of the matrix A and rank computation for controllability matrix (6) realized on the basis of singular value decomposition in modern mathematical software tools [10]:

$$(21) \quad N_{\text{Kalman}} \approx 2(n-r-1)n^3 + 2n((n-r+1)r)^2 + 2((n-r+1)r)^3.$$

In general case, the estimation of computation cost for controllability recursive test can not be easily obtained because of considerable dependence of dimensions of matrices (9) and the number of test steps on the system structure. However, the results of carried out computation experiments show, that with increase of n and decrease of difference $(n-r)$, the total computation cost strives for sum of costs caused by determination of zero divisors of the initial matrix B and the matrix in equation (7) on the first step. Thus, taking into account that in modern mathematical software tools computation of zero divisors (null-spaces) of a matrix is realized using economical singular value decomposition, we can write [10]

$$(22) \quad N_{recursive} \approx 6nr^2 + 11r^3 + 12(n-r)n^2 + 11r^3 = 12n^3 + 22r^3 + 6nr^2 - 12rn^2.$$

Figure 1 presents the relationships between operations' number ratio $N_{Kalman} / N_{recursive}$ and r / n for the matrices A and B with elements given by random values from Gaussian random sequence for $n = 50$, $n = 100$, and $n = 200$.

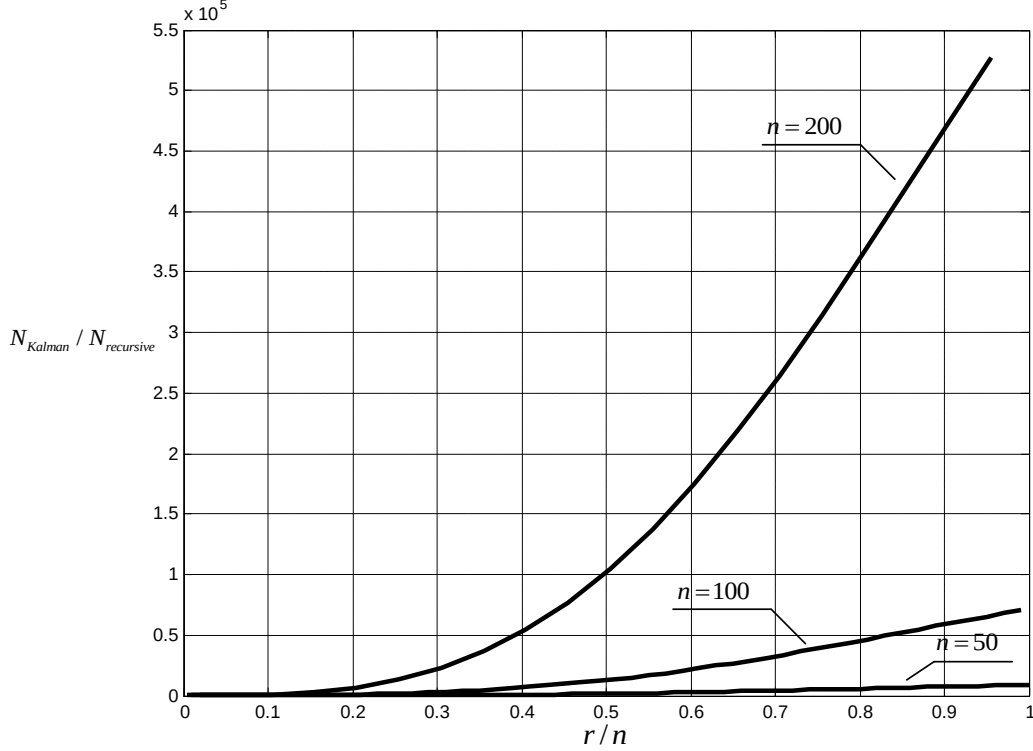


Fig. 1: Relationships between operations' number ratio $N_{Kalman} / N_{recursive}$ and r / n

Figure 2 presents the diagrams of required CPU-time for use of the recursive test and Kalman controllability criterion.

Analyzing expressions (21), (22) and computation results presented in Figures 1 and 2, one can make a conclusion that with increase of system dimension n and ratio r/n , the recursive controllability test for power system allows significant reduction of the computation cost.

Computation cost estimation for the system controllability test can be carried out similarly to the technique described above.

7 CONCLUSION

The recursive tests developed on the basis of the controllability and observability band criteria have the simple form and can be easily realized in computing environment. These tests allow significant reduction of computation cost for power system controllability and observability analysis. At present time, the works on application of the recursive controllability tests to real-time analysis of controllability and observability of consolidated electric power system of Center of Russia being a part of complex automated system of technological control of Main Power Network of Center. These works are carried out as a pilot project on smart grid JSC «FGC UES».

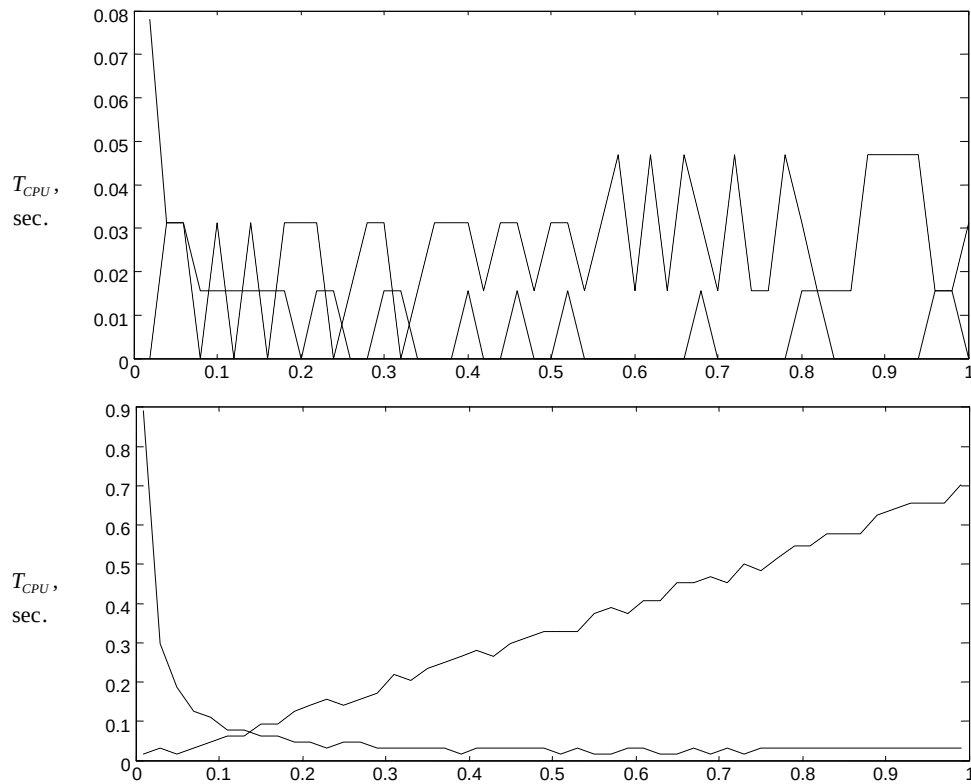


Fig. 2: Required CPU-time while using the recursive test (solid line) and Kalman controllability criterion (dotted line)

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